→ **Fraction**

Rational Expression

$$\frac{p(x)}{q(x)}, q \neq 0$$

Examples of Rational Functions

$$-\frac{13}{42} \qquad \frac{7y}{8z} \qquad \frac{5x+2}{x^2-7} \qquad \frac{4x^2+3x-1}{2x-8}$$

Undefined Values –

What makes the bottom zero

$$\frac{X+3}{7}$$

Determine the values for which the rational expression is undefined

$$\frac{9y}{x} \qquad\qquad \frac{4b-3}{2b+5} \qquad\qquad \frac{x+4}{x^2+5x+6}$$

$X = 0$

$2b+5=0$
$2b=-5$
$b=-\frac{5}{2}$

$x^2+5x+6=0$
$(x+2)(x+3)=0$
$x+2=0 \quad x+3=0$
$x=-2 \quad x=-3$

Evaluate $\frac{y+1}{2y-3}$ for each value:

$y = 1$

$$\frac{1+1}{2(1)-3}$$
$$\frac{2}{-1}=-2$$

$y = -3$

$$\frac{-3+1}{2(-3)-3}$$
$$\frac{-2}{-9}=\frac{2}{9}$$

$y = 0$

$$\frac{0+1}{2(0)-3}$$
$$\frac{1}{-3}=-\frac{1}{3}$$

Evaluate $\frac{x^2+8x+7}{x^2-4}$ for each value:

$x = 0$

$$\frac{0^2+8(0)+7}{(0)^2-4}$$
$$\frac{7}{-4}$$

$x = 2$

$$\frac{(2)^2+8(2)+7}{(2)^2-4}$$
$$\frac{27}{0}$$
Undefined

$x = -1$

$$\frac{(-1)^2+8(-1)+7}{(-1)^2-4}$$
$$\frac{1-8+7}{-3}=\frac{0}{-3}=0$$

Simplified Rational Expression	Simplify:		
A Rational expression is considered simplified if there are not common factors in its numerator and denominator.	$-\dfrac{36}{63} = -\dfrac{4}{7}$	$\dfrac{45}{81} = \dfrac{5}{9}$	$-\dfrac{42}{54} = -\dfrac{7}{9}$

Equivalent Fractions Property

If a, b and c, are numbers where $b \neq 0$, and $c \neq 0$, then

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c} \quad and \quad \frac{a \cdot c}{b \cdot c} = \frac{a}{b}$$

	$\dfrac{3xy}{18x^2y^2}$	$\dfrac{4x^2y}{12xy^2}$	$\dfrac{16x^2y}{2x^2y^2} = \dfrac{8}{y}$
	$\dfrac{1}{6xy}$	$\dfrac{1x}{3y}$	
		$\dfrac{x}{3y}$	

Simplify a Rational Expression

Step 1 - Factor the numerator and denominator completely

Step 2 – Simplify by dividing out common factors.

We can only
divide out (cancel)
Factors not terms ↑
multiply

$(p^3 - p^2) + (2p - 4)$
$p^2(p-1) + 2(p-$

Simplify:

$\dfrac{2x+8}{5x+20}$	$\dfrac{3x-6}{2x-4}$	$\dfrac{7y+35}{5y+25} = \dfrac{7(y+5)}{5(y+5)} = \dfrac{7}{5}$
$\dfrac{2(x+4)}{5(x+4)} = \dfrac{2}{5}$	$\dfrac{3(x-2)}{2(x-2)}$	
	$\dfrac{3}{2}$	
$\dfrac{x^2+5x+6}{x^2+8x+12}$	$\dfrac{x^2-x-2}{x^2-3x+2}$	$\dfrac{x^2-3x-10}{x^2+x-2}$: $\dfrac{(x-5)(x+2)}{(x+2)(x-1)}$
$\dfrac{(x+3)(x+2)}{(x+2)(x+6)}$	$\dfrac{(x-2)(x+1)}{(x-2)(x-1)}$	$\dfrac{x-5}{x-1}$
$\dfrac{x+3}{x+6}$	$\dfrac{x+1}{x-1}$	
$\dfrac{y^2+y-42}{y^2-36}$	$\dfrac{x^2+x-6}{x^2-4}$	$\dfrac{p^3-p^2+2p-4}{p^2-7p+10}$
$\dfrac{(y+7)(y-6)}{(y+6)(y-6)}$	$\dfrac{(x+3)(x-2)}{(x-2)(x+2)}$	$\dfrac{p^3-p^2+2p-4}{(p-5)(p-2)}$
$\dfrac{y+7}{y+6}$	$\dfrac{x+3}{x+2}$	
$\dfrac{2n^2-14n}{4n^2-16n-48}$	$\dfrac{3b^2-12b+12}{6b^2-24}$	$\dfrac{2x^2-12x+18}{3x^2-27}$
$\dfrac{2n(n-7)}{4(n^2-4n-12)}$	$\dfrac{3(b^2-4b+4)}{6(b^2-4)}$	$\dfrac{2(x^2-6x+9)}{3(x^2-9)}$
$\dfrac{2n(n-7)}{4(n-6)(n-2)}$	$\dfrac{3(b-2)(b-2)}{6(b-2)(b+2)}$	$\dfrac{2(x-3)(x-3)}{3(x-3)(x+3)}$
$\dfrac{n(n-7)}{2(n-6)(n-2)}$	$\dfrac{b-2}{2(b+2)}$	$\dfrac{2(x-3)}{3(x+3)}$

Rational Expressions with opposite Factors $X+8 = 8+X$	(math work below)

$$\frac{m^3+8}{m^2-4}$$

$$\frac{(m+2)(m^2-2m+4)}{(m-2)(m+2)}$$

$$\frac{m^2-2m+4}{m-2}$$

$$\frac{p^3-64}{p^2-16} \quad \frac{(p-4)(p^2+4p+16)}{(p-4)(p+4)}$$

$$\frac{p^2+4p+16}{p+4}$$

$$\frac{x^3+8}{x^2-4} \quad \frac{(x+2)(x^2-2x+4)}{(x+2)(x-2)}$$

$$\frac{x^2-2x+4}{x-2}$$

$$\frac{x-8}{8-x} = \frac{x-8}{-x+8}$$

$$\frac{x-8}{-1(x-8)}$$

$$= -1$$

$$\frac{14-2x}{x^2-49} = \frac{-2x+14}{x^2-49}$$

$$\frac{-2(x-7)}{(x-7)(x+7)}$$

$$\frac{-2}{x+7}$$

$$\frac{3y-27}{81-y^2} = \frac{3y-27}{-y^2+81}$$

$$\frac{3y-27}{-(y^2-81)}$$

$$\frac{3(y-9)}{-(y-9)(y+9)}$$

$$-\frac{3}{(y+9)}$$

$$\frac{x^2-4x-32}{64-x^2}$$

$$\frac{(x-8)(x+4)}{-(x^2-64)}$$

$$\frac{(x-8)(x+4)}{-(x+8)(x-8)}$$

$$-\frac{x+4}{x+8}$$

$$\frac{4-x^2}{x^3-8}$$

$$\frac{-x^2+4}{x^3-8}$$

$$\frac{-(x^2-4)}{x^3-8}$$

$$\frac{-(x-2)(x+2)}{(x-2)(x^2+2x+4)}$$

$$\frac{-(x+2)}{x^2+2x+4}$$

Multiply a rational expression
Step 1 – Factor each numerator
and denominator completely

Step 2 – Multiply the
numerators and denominators

Step 3 – Simplify by dividing out
common factors

(handwritten:)
$$4$$
$$28$$
$$15$$
$$148$$
$$280$$
$$0$$

Multiply:

$\dfrac{10}{28} \cdot \dfrac{8}{15} = \dfrac{80}{420}$

$= \dfrac{8}{42}$

$= \dfrac{4}{21}$

$\dfrac{6}{10} \cdot \dfrac{15}{12} = \dfrac{90}{120}$

$= \dfrac{3}{4}$

(handwritten:) $\dfrac{20}{18} \cdot \dfrac{6}{5} = 1$

$\dfrac{2x}{3y^2} \cdot \dfrac{6xy^3}{x^2y}$

$\dfrac{12x^2y^3}{3x^2y^3}$

4

$\dfrac{3pq}{q^2} \cdot \dfrac{5p^2q}{6pq}$

$\dfrac{15p^3q^2}{6pq^3}$

$\dfrac{5p^2}{2q}$

$\dfrac{6x^3y}{7x^2} \cdot \dfrac{2xy^3}{x^2y}$

$\dfrac{12x^4y^4}{7x^4y} = \dfrac{12y^3}{7}$

$\dfrac{2x}{x^2-7x+12} \cdot \dfrac{x^2-9}{6x^2}$

$\dfrac{5x}{x^2+5x+6} \cdot \dfrac{x^2-4}{10x}$

$\dfrac{n^2-7n}{n^2+2n+1} \cdot \dfrac{n+1}{2n}$

$\dfrac{x^2-25}{x^2-3x-10} \cdot \dfrac{x+2}{x}$

$\dfrac{16-4x}{2x-12} \cdot \dfrac{x^2-5x-6}{x^2-16}$

$\dfrac{12x-6x^2}{x^2+8x} \cdot \dfrac{x^2+11x+24}{x^2-4}$

$\dfrac{9v-3v^2}{9v+36} \cdot \dfrac{v^2+7v+12}{v^2-9}$

$\dfrac{3a-21}{a^2-9a+14} \cdot \dfrac{a^2-4}{3a+6}$

$\dfrac{b^2-b}{b^2+9b-10} \cdot \dfrac{b^2-100}{b^2-10b}$